My Reminiscences of Trygve Haavelmo at the Cowles Commission

T. W. Anderson*
Stanford University

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1 Introduction

Trygve Haavelmo was my colleague at the Cowles Commission for Research in Economics at the University of Chicago for a large part of the academic year 1945-46. Since then, we have met frequently in Oslo. Much of my research and writing was stimulated by his path-breaking work, particularly "The probability approach in econometrics," 1944.

My doctoral dissertation in mathematics at Princeton University was accepted in June, 1945; World War II was coming to an end in that summer. For several years I had been doing war research (evaluation of weather forecasting methods, analysis of battleship gunfire battles, and sequential methods of sensitivity testing). I was looking forward to getting back to full time academic pursuits: teaching and basic research. I tentatively accepted an instructorship in mathematics at Princeton for the academic year 1945-46 which started November 1st. However, it was understood that I would be excused from this obligation if I was offered a position that was more interesting to me. Because I was interested in statistics and economics, Oskar Morgenstern suggested I talk with Arthur Burns, Director of the National Bureau of Economic Research in New York City. At the end of my interview with Burns he said "The National Bureau has no need for someone as highly trained in statistics as you."

*Department of Statistics and Department of Economics, Stanford University, Stanford, CA 94305, USA; `twa@stanford.edu`. The author thanks Olav Björkholt for assistance in providing much of the historical material in this paper.

In October Morgenstern suggested that I talk with Jacob Marschak, who was the director of the Cowles Commission for Research in Economics at the University of Chicago. Marschak had been at a conference on Peaceful Uses of Atomic Energy at Rye, NY (a suburb of New York City), where I met him. Marschak told me about the activities at the Cowles Commission, largely continuing Haavelmo's work on simultaneous equation models. I told him about my research in multivariate statistics including what came later to be known as "reduced rank regression." We both saw that there was a good fit between our interests.

Marschak told me that he would like to offer me a position at the Cowles Commission, but he would like to confer with his colleague Tjalling Koopmans first. He was taking the overnight train to Chicago and would talk to Koopmans the first thing in the morning. I told him that that was not satisfactory. The next day was Monday, November 1st, which was the first day of the fall semester at Princeton. I was free to accept any position I wanted before November 1, but on November 1 I was committed to teaching mathematics at Princeton. Marschak decided to offer me a position before he boarded the train. Within three weeks I was in Chicago. I left the Cowles Commission in mid-September of 1946 to become an instructor of mathematical statistics at Columbia University.

2 Life at the Cowles Commission

When I arrived at the Cowles Commission (the "CC"), I found that Koopmans, Roy Leipnik, and Herman Rubin were studying the use of maximum likelihood methods in simultaneous equations models. I knew Koopmans from his earlier visit to Princeton when he was interested in questions of serial correlation. Leipnik and Rubin were graduate students in mathematics at the University of Chicago.

Much of the research at the CC during 1945-46 was reported in *Statistical Inference in Dynamic Economic Models* (T.C. Koopmans, editor), 1950. It included a paper by Trygve, "Remarks on Frisch's confluence analysis and its use in econometrics." This paper put confluence analysis in terms of suitable probability models although Frisch had not used this language.

In January of 1946 there was a small conference at the CC that included M.A. ("Abe") Girshick, then of the US Department of Agriculture. I knew him because we had independently developed the mathematics of the non-central Wishart distribution several years before (Anderson and Girshick, 1944). Haavelmo arrived at the CC about the same time as I. Girshick presented an example of a two-equation model of two endogenous variables and two exogenous variables. Each equation included one exogenous variable; each equation was "just" identified.

It was clear that statistical methods were needed for structural equations that were over-identified. I was prepared for that issue because of my dissertation on estimation of regression models (reduced forms) of arbitrary rank. Once the reduced form (or the relevant part of it) was estimated with the desired rank, the algebra of the "just identified" equation can be applied. The method can be framed as maximum likelihood or as multivariate analysis of variance; see Anderson and Rubin (1949, 1950).

Koopmans decided to call the application of maximum likelihood to the entire set of simultaneous equations as "Full Information Maximum Likelihood" and the development of the application to a single equation as "Limited Information Maximum Likelihood." The latter is often abbreviated to a single syllable: LIML.

Questions of notation and terminology were treated as very important. I hate to think of the amount of time that was spent on the question of whether to use row vectors or column vectors. We thought we should have a uniform notation as well as a uniform terminology.

The entire CC was housed in one huge room except for Marschak, the director, and Dickson Leavens, the business manager, who were in separate offices down the hall. Along the left side of the main room were four cubicles occupied by Koopmans, Leipnik, Rubin, and Lawrence Klein. Along the right side were cubicles for Leonid Hurwicz and me. The central space was used for seminars and had desks for latecomers, one of which was for Haavelmo and another for Sonia Adelman (later to become Mrs. Klein).

Because this room faced south it was a pleasant space. The partitions were only seven feet high in a room with twenty foot-high ceilings. However, some of the occupants had strong voices and some liked to talk a lot. The partitions had to be raised to the ceiling.

The set-up encouraged communication among the occupants and that was a good thing.

Discussions and seminars were convened on short notice. Haavelmo joined many of them. A regular seminar was scheduled for each week; one member took minutes. What sometimes was a problem was that we were not all on the same time schedule. Haavelmo was quite independent; he interacted with many of us, probably more with Klein than with others. Klein was focusing on constructing a full model of the US economy.

Herman Rubin joined me in developing the LIML estimation of the coefficients of a single equation. We developed the procedure and presented it at a meeting of statisticians at the end of the summer of 1946 (Institute of Mathematical Statistics, August 23, 1946, Ithaca, New York). It took longer to analyze the asymptotic distribution theory.

3 Statistical analysis of the demand for food

A major interest of Haavelmo while he was at the CC was the application of the "probability approach" to the analysis of demand for agricultural products, particularly food. Haavelmo devoted several CC seminars to this subject (March 19 and November 21). One aspect of his activity was to demonstrate his approach; another purpose was to reach substantive conclusions; a third objective was to illustrate the use of the LIML estimation procedure. Approximately one-half of the paper was devoted to carrying out the calculations of the LIML estimate for one version of the model described below. (Note the second half of the title of the paper.) Anderson and Rubin could have been joint authors.

Girshick and Haavelmo collaborated on the study of quantitative research in agricultural economics sponsored by the CC and the Department of Economics of the University of Chicago. They reported this study in a 1947 *Econometrica* paper, "Statistical analysis of the demand for food: Example of simultaneous estimation of structural equations." Haavelmo spent two weeks in April and May, 1946, in Washington working with Girshick on this paper. They developed a model of five equations for five economic (dependent or endogenous) variables. The equations were

A. The demand for food.

B. The income equation.

C. Supply of food in the retail market.

D. Supply of foodstuffs by farmers.

E. The demand for farm food products by the commercial sector.

The jointly dependent variables were (1) food consumption, (2) food prices, (3) disposable income, (4) food production, and (5) prices received by farmers for food. These five equations for five dependent variables form a "complete system." The "predetermined" variables (also known as "exogenous variables") were (1) prices received by farmers lagged one year, (2) net investment, (3) time, and (4) disposable income lagged one year. The "reduced form" expressed the dependent variables as functions of the predetermined variables plus unobserved random variables. There were 20 observations (that is, 20 years of annual data). Every equation was identified; some were over-identified.

Since this model is *complete* in the sense that the set of five structured equations is equal to the number of jointly dependent variables, Girshick and Haavelmo had the choice of estimating the coefficients of the equations by Full Information Maximum Likelihood (FIML) or by Limited Information Maximum Likelihood (LIML), equation by equation. They chose the latter. One reason for the choice was that FIML required heavy computation and the numerical methods were still being developed. Another reason was that the methodology for LIML was fully developed and was therefore available to other econometricians to use. A further advantage of the use of the LIML technology is that one particular structural equation can be modified without affecting the calculations for the other structural equations.

The paper gives all of the data needed. The reduced form was estimated as a multivariate regression. Each dependent variable was a linear function of the independent variables plus an unobserved disturbance.

The raw data were refined by deflation by price indices, conversion to per capita basis, and other adjustments. One of the predetermined variables is a dependent variable lagged one year; thus the model is a first-order autoregressive model. The LIML procedure uses the eigenvalues and eigenvectors of submatrices of the constituent matrices; these were computed numerically (matrices of order two or three). The computations were summarized fully, making it easy for the reader to follow. Only desktop computers were needed. These procedures were developed by Anderson and Rubin.

4 The marginal propensity to consume

One of the simplest examples of the statistical study of a simultaneous equations model involves three observable variables:

1. c_t = consumers' expenditure;
2. y_t = disposable income;
3. z_t = investment expenditure.

One relationship between these three variables is the identity

$$y_t = c_t + z_t.$$

A structural or behavioral equation is the "consumption function"

$$c_t = \alpha y_t + \beta + u_t,$$

where u_t is an unobservable disturbance. Third, z_t is "autonomous," that is, it is determined independently. It may be a sequence of given numbers or it may be a random variable stochastically independent of u_t. From the identity and the consumption function, we can derive a "reduced form"

$$\begin{aligned} c_t &= \frac{\alpha}{1-\alpha} z_t + \frac{\beta}{1-\alpha} + \frac{u_t}{1-\alpha}, \\ y_t &= \frac{1}{1-\alpha} z_t + \frac{\beta}{1-\alpha} + \frac{u_t}{1-\alpha}; \end{aligned}$$

each of these is a regression. From either of these two equations one can derive consistent estimates of $\alpha/(1-\alpha)$, $\beta/(1-\alpha)$, and $1/(1-\alpha)$ and hence consistent estimates of α and β. However, if the parameters of the consumption function are estimated by the regression of c_t on y_t, the estimates are not consistent (because u_t is not independent of y_t).

Haavelmo goes on to consider elaborations of this model. For example, he considers a model in which investment z_t is the sum of an autonomous part and a part depending linearly on income y_t.

This paper was completed in June of 1946 and submitted to *JASA*. A footnote acknowledges my pointing out that a confidence region for two parameters can be obtained in terms of Hotelling's T^2.

5 Cowles Commission Conference, September 19 to 21, 1946

Before I left for Columbia in late September, there was a conference at the CC to discuss many of the statistical problems of simultaneous equation models that had arisen since I arrived at the CC in November 1945. The participants included Kenneth Arrow, Harold Davis, Olav Reiersol, Gerhard Tintner, and Allen Wallis, among others. Haavelmo had returned to Oslo to defend his doctoral thesis (Haavelmo, 1944). The key topic was the LIML estimation procedures (also called "reduced form method") and related problems such as identification, small-sample confidence regions, medium-sized sample confidence regions, aggregation, and testing multiple hypotheses. Most of the second day was spent on the "reduced form method" (LIML).

In building a model the econometrician may start with a set of G structural or behavior equations such as items A–E in Section 3. These equations can be solved for the G economic or dependent variables. The relevant data can be summarized as a $T \times G$ matrix $\boldsymbol{Y}$ of G dependent (economic) variables observed over T time points and a $T \times K$ matrix $\boldsymbol{Z}$ of K independent variables. In Girshick and Haavelmo (1947) $G = 5,\ K = 4$, and $T = 20$. The reduced form is a multivariate regression

$$\boldsymbol{Y} = \boldsymbol{Z}\boldsymbol{\Pi} + \boldsymbol{V},$$

where $\boldsymbol{\Pi}$ is a $K \times G$ matrix of parameters and $\boldsymbol{V}$ is a $T \times G$ matrix of (unobservable) disturbances. The rows of $\boldsymbol{V}$ are independent normal vectors with covariance matrix $\boldsymbol{\Omega}$.

The first step of the Anderson–Rubin procedure is the estimate $\boldsymbol{\Pi}$ by the regression of $\boldsymbol{Y}$ on $\boldsymbol{Z}$,

$$\boldsymbol{P} = (\boldsymbol{Z}'\boldsymbol{Z})^{-1}\boldsymbol{Z}'\boldsymbol{Y}.$$

The sample covariance matrix of the residuals $\boldsymbol{Y} - \boldsymbol{Z}\boldsymbol{P}$ estimates the covariance matrix $\boldsymbol{\Omega}$.

A particular structural equation may involve a subset of $\boldsymbol{Y}$ and a subset of $\boldsymbol{Z}$

In each structural equation there are some y_{gt} depending on some z_{kt}. The LIML method of estimation is to find the linear combination of y's with smallest variance after removing the effect of the z's excluded from that structural equation. The method is a kind of multivariate analysis of variance. The computations were described completely in Girshick and Haavelmo (1947); the mathematical development was reported in Anderson and Rubin (1949).

Another approach to inference in simultaneous equation models is to assume that the equations hold for "true values" of the variables, but these variables are subject to "errors of measurement." This formulation is more like the approach of Ragnar Frisch in his *Confluence Analysis*, 1934. Haavelmo had been a student of Frisch as well as an assistant. A formulation of this model was given by Gerhard Tintner in 1946.

Leo Hurwicz and I compared the Haavelmo approach (which we called "shocks in equations") with the Tintner model (which we called "errors in variables"). Hurwicz had studied at the London School of Economics and the Massachusetts Institute of Technology. He had been a member of the Institute of Meteorology at the University of Chicago from 1942 to 1944. He presented our paper "Errors and shocks in economic relationships" at the Econometric Society in Washington, DC, on September 8, 1947.

6 Other activities

Haavelmo returned to Oslo in September. He gave one "trial lecture" on September 28 and a second trial lecture September 30. The defense of his doctoral dissertation, "The probability approach in econometrics," took place October 1. The examiners ("opponents") were Ragnar Frisch and Lange Nielsen. After his dissertation was accepted Haavelmo returned to Chicago.

One of the recurring problems was how to compute the quantities needed for analysis. John von Neumann, who knew Marschak from Berlin or Vienna, occasionally visited the CC on his trips between Princeton and the West. Each time we asked him for suggestions. His usual answer was that at Princeton they were developing a powerful computer; when that was finished he would answer our questions. It is ironic that for typical models in practice (a small number of endogenous variables in an equation) the LIML method is still the preferred method. Enhanced computing power has not changed that fact.

The university organized weekly lectures on the important developments in the war years, such as Frank Lloyd Wright on architecture and Schöenberg on music; von Neumann gave the lecture on mathematics. In view of Kurt Gödel's results on consistency and solvability, von Neumann expressed the view that mathematics was an experimental science, in the sense that no contradiction so far has been found.

7 Three Sailors

Trygve, Larry and I thought that we should exploit the proximity of the CC to Lake Michigan. Accordingly we bought a used sailboat of modest size. We spent many evenings and weekends putting our boat in shape. Made of wood, it required stripping the paint, sanding the hull, and repainting it. Finally it was ready! Since we could not get a mooring near the CC, we got one downtown near the Navy Pier and Grant Park. It was really a harbor for large yachts. We needed a dinghy (a small row boat) to get from the shore to the mooring. Trygve and Larry were delighted to find a dinghy made by KARL MARX.

I got a friend who was a veteran sailor to help me get our boat from the garage near the CC to a boat entry to our mooring. One fine day Larry and I proceeded to the harbor and took the dinghy to our boat. Neither of us had sailed a boat, but we understood the physics of it. We started raising the sails, but neither of us had tied the dinghy to our boat. To our horror we saw our dinghy being blown by a west wind towards the open water of Lake Michigan.

We hurriedly finished putting up the sails and headed after the dinghy. The dinghy was being blown towards the space between the arms of the breakwaters of the harbor. It did not take us long to realize that not only was the dinghy being blown toward Michigan, but we were, too. We gave up the dinghy, and looked to save ourselves. We tacked back and forth, but still moved eastward. Finally I went to the bow of our boat and grabbed on to a boat moored near the breakwater. We simply waited for the harbor master in his power boat to pick us up and take us to our mooring. After we took down our sails and tied our boat to its mooring, the harbor master took us to shore.

Several days later Trygve told us that he was by the harbor the previous evening and could not see our boat. The three of us proceeded to the harbor. We were dismayed to see that our boat was moored, but the mast was broken and was tied to the hull. We learned that in a storm with an east wind our boat was blown off its mooring and onto the rocks lining the harbor. At that point we gave up our dreams of sailing Lake Michigan. Sonia, Dorothy, and Beulah were very disappointed; they never got on our boat.

8 Postscript

The LIML or "reduced form" method of estimating the coefficients of a structural equation was developed as a maximum likelihood procedure. In many problems the likelihood approach has some desirable small-sample properties as well as optimal asymptotic properties. The LIML method also has an interpretation in terms of the multivariate analysis of variance.

Anderson, Stein, and Zaman (1985) have shown that the LIMLK estimator is admissible when the LIMLK estimator is the LIML estimator with the sample covariance matrix replaced by the assumed model covariance matrix. The September 1946 conference and Girshick–Haavelmo show that the LIML estimator was feasible; that is, computational procedures were available. Why has the econometric community been so reluctant to accept them? See Anderson (1991, 2005) for more history.

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